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On spaces with k-in-countable bases or weak bases ☆

Yu Zuoming [a], Yun Ziqiu [b,*]

[a] School of Zhangjiagang, Jiangsu University of Science and Technology, Zhangjiagang, 215600 PR China
[b] Department of Mathematics, Soochow University, 215006 PR China

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ABSTRACT

We prove that, for any $k \in N$, every regular star compact space with a k-in-countable base is metrizable. We also provide a metrization theorem for compact spaces with 2-in-finite weak bases; this gives a partial answer to a question of Bennett and Martin. It turns out that, for any $m \in N$, if X has an m-in-countable base and weak countable tightness (or k-property), then X is strongly monotonically monolithic. We apply this result to show that an example of Davis, Reed and Wage provides a consistent negative answer to a problem of Tkachuk.

1. Introduction

It is proved in [1, Theorem 15] that every countably compact regular space with a k-in-countable base for some $k \in N$ is metrizable. Meanwhile, there exist pseudocompact spaces with point-countable bases which are not normal, hence not metrizable. As star compactness is a property between countable compactness and pseudocompactness [11], the following question is natural:

Question 1.1. Is every regular star compact space with a k-in-countable base metrizable?

We give a positive answer to Question 1.1.

Hoshina proved that every compact Hausdorff space with a point-countable weak base is metrizable. Bennett and Martin raised the following question in [2]:

Question 1.2. Is every compact Hausdorff space with a 2-in-finite weak base metrizable?

In this note, we give a partial answer to this question by proving the following result:

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* Corresponding author.
E-mail addresses: yuzuoming1981@gmail.com (Z. Yu), yunziqiu@public1.sz.js.cn (Z. Yun).

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Every compact Hausdorff space X of size $< \mathfrak{b}$ and with a 2-in-finite weak base $\mathcal{B}$ consisting of sequentially closed subsets of X is metrizable.

Tkachuk formulated the following question (see [7, Question 3.4]):

Question 1.3. Does every strongly monotonically monolithic space have a point-countable base?

In this note, we give a sufficient condition for a space with a k-in-countable base to be strongly monotonically monolithic. Applying this result we show that an example constructed by Davis, Reed and Wage consistently answers Question 1.3.

2. Notation and terminology

All the spaces in this paper are at least Hausdorff and N denotes the set of positive integers.

Given a natural number k, a family $\mathcal{A}$ of subsets of a space X is called k-in-countable if every set $A \subseteq X$ with $|A| = k$ is contained in at most countably many elements of $\mathcal{A}$. The family $\mathcal{A}$ is 2-in-finite if every two-point subset of X is contained in only finitely many elements of $\mathcal{A}$. 2-in-finite base was called *weakly uniform base* in [3,4], and was called *pair-finite base* in [2]. But in this note, we only use the term 2-in-finite base. The concept of 2-in-finite base is useful in resolving certain unsolved problems regarding spaces with a G_δ-diagonal since Hausdorff spaces with a 2-in-finite base have a G_δ-diagonal [4].

Let $\mathcal{P}$ be a class (or a property) of a space X. The space X is said to be *star* $\mathcal{P}$ if for any open cover $\mathcal{U}$ of X there is a subspace $Y \in \mathcal{P}$ of X such that $st(Y, \mathcal{U}) = X$, where $st(Y, \mathcal{U}) = \bigcup\{U \in \mathcal{U}\colon U \cap Y \neq \emptyset\}$ [6].

Let A be a subset of a space X. A family $\mathcal{B}$ of subsets of X is called an *external network* (*base*) of A in X if (all elements of $\mathcal{B}$ are open in X and) for any $x \in A$ and any open set U in X with $x \in U$, there exists some $B \in \mathcal{B}$ such that $x \in B \subset U$.

We say that a space X is *monotonically monolithic* (*strongly monotonically monolithic*) [7] if, for each $A \subset X$ we can assign an external network (base) $\mathcal{O}(A)$ to the set $\overline{A}$ satisfying the following conditions:

(a) $|\mathcal{O}(A)| \leqslant \max\{|A|, \omega\}$;
(b) if $A \subset B$, then $\mathcal{O}(A) \subset \mathcal{O}(B)$;
(c) if α is an ordinal and we have a family $\{A_\beta\colon \beta < \alpha\}$ of subsets of X such that $\beta < \beta'$ implies $A_\beta \subset A_{\beta'}$, then $\mathcal{O}(\bigcup_{\beta<\alpha} A_\beta) = \bigcup_{\beta<\alpha} \mathcal{O}(A_\beta)$.

Assume that X is a space and $x \in X$. We say that X is *weakly countably tight* at x if there is a countable subset A of $X \setminus \{x\}$ such that x is in the closure of A. A space X is called *weakly countably tight* if X is weakly countably tight at every $x \in X$ [1].

The "*extent*" $e(X)$ of a topological space X is the supremum of the cardinalities of closed discrete subsets of X. The *Lindelöf number* $l(X)$ of X is the smallest cardinal λ such that every open cover of X has a subcover the cardinality of which is at most λ.

A *weak base* for a topological space is a collection $\mathcal{P} = \bigcup\{\mathcal{P}_x\colon x \in X\}$ of subsets of X such that

(1) for each x, the family $\mathcal{P}_x$ is closed under finite intersections and $x \in \bigcap \mathcal{P}_x$, and
(2) a subset U of X is open if and only if for each $x \in U$, there is a $P \in \mathcal{P}_x$ such that $x \in P \subset U$.

Let $\mathcal{P} = \bigcup\{\mathcal{P}_x\colon x \in X\}$ be a weak base on X. A subset U of X is called a *weak neighborhood* of x if there is a $P \in \mathcal{P}_x$ such that $x \in P \subseteq U$. The space X is said to be *weakly first countable* if $\mathcal{P}_x$ is a countable collection for each $x \in X$.

3. Metrizability of spaces with *k*-in-countable bases

The following Lemmas 3.1 and 3.2 will be used frequently in the sequel.

Lemma 3.1. *([1, Lemma 14]) Let $\mathcal{B}$ be a k-in-countable base of a space X for some $k \in N$. Then $\mathcal{B}$ is point-countable at every non-isolated point of X at which X is weakly countably tight.*

Lemma 3.2. *([1, Theorem 15]) Every countably compact regular space with a k-in-countable base for some $k \in N$ is metrizable.*

Lemma 3.3. *If X is a space with a k-in-countable base for some $k \in N$, then $e(X) = l(X)$.*

Proof. We only need to prove that $e(X) \geqslant l(X)$ since $e(X) \leqslant l(X)$ holds for every space. Because it is trivially true if $l(X) \leqslant \omega$, we can assume that $l(X)$ is uncountable.

Let $\mathcal{U}$ be an open cover of X and let κ be a cardinal such that there is no subfamily of $\mathcal{U}$ of size less than κ that covers X. We prove that there is a closed discrete subset D of X with $|D| = \kappa$.

Assume that $\mathcal{B}$ is a k-in-countable base of X for some $k \in N$. Let $\mathcal{B}'$ be a subcollection of $\mathcal{B}$ such that $\mathcal{B}'$ refines $\mathcal{U}$. For each finite subset H of X with $|H| = k$, if there is no $B \in \mathcal{B}$ such that $H \subset B$, then fix some finite subcollection $\mathcal{B}_H \subseteq \mathcal{B}'$ such that $H \subset \bigcup \mathcal{B}_H$ and let $\mathcal{C}(H) = \mathcal{B}_H$; otherwise, let $\mathcal{C}(H) = \{B \in \mathcal{B}' \colon H \subset B\}$. Let $\mathcal{C}(X) = \{\bigcup \mathcal{C}(H) \colon H$ is a finite subset of X with $|H| = k\}$. There is no subfamily of $\mathcal{C}(X)$ of size less than κ that covers X by the choice of $\mathcal{U}$ and the assumption that $\mathcal{B}$ is k-in-countable.

Pick $x_0, x_1, \dots, x_{k-1} \in X$ arbitrarily. Let $k \leqslant \alpha < \kappa$. Suppose that for each $\beta < \alpha$, a point x_β has been defined. Then we can pick $x_\alpha \in X \setminus \bigcup(\bigcup\{\mathcal{C}(H) \colon H$ is a finite subset of $\{x_\beta \colon \beta < \alpha\}$ with $|H| = k\})$ since $|\{\bigcup \mathcal{C}(H) \colon H$ is a finite subset of $\{x_\beta \colon \beta < \alpha\}$ with $|H| = k\}| < \kappa$.

Let $D = \{x_\alpha \colon \alpha < \kappa\}$. To prove that D is closed and discrete, we show that no point in X is a cluster point of D.

Otherwise, we can pick a point y in X such that y is a cluster point of D. Let β_y be the least ordinal such that y a cluster point of $\{x_\alpha \colon \alpha < \beta_y\}$. There is some finite subset H of $\{x_\alpha \colon \alpha < \beta_y\}$ with $|H| = k$ such that $y \in \bigcup \mathcal{C}(H)$. In fact, we can choose some $B \in \mathcal{B}'$ such that $y \in B$ since $\mathcal{B}'$ is a cover of X. By $y \in \overline{\{x_\alpha \colon \alpha < \beta_y\}}$, we obtain that $|B \cap \{x_\alpha \colon \alpha < \beta_y\}| \geqslant \omega$. Pick a finite set $F \subset B \cap \{x_\alpha \colon \alpha < \beta_y\}$ such that $|F| = k$. Then $y \in \bigcup \mathcal{C}(F)$ by the construction of $\mathcal{C}(F)$. Let $\alpha_y = \max\{\alpha \colon x_\alpha \in F\}$. $y \notin \overline{\{x_\alpha \colon \alpha > \alpha_y\}}$ since $\{x_\alpha \colon \alpha > \alpha_y\} \subset X \setminus \bigcup \mathcal{C}(F)$. $y \notin \overline{\{x_\alpha \colon \alpha < \alpha_y\}}$ since $\alpha_y \geqslant \beta_y$. Therefore, $X \setminus (\overline{\{x_\alpha \colon \alpha > \alpha_y\}} \cup \overline{\{x_\alpha \colon \alpha < \alpha_y\}} \cup \{x_{\alpha_y}\})$ is an open neighborhood of y which contains no element of D. This contradicts the fact that y is a cluster point of D. Therefore, y is not a cluster point of D. Thus, there does not exist cluster point of D in X. It follows that D is closed and discrete.

Since the family $\mathcal{U}$ was taken arbitrarily, we can see that $e(X) \geqslant l(X)$. □

Theorem 3.4. *If X is a regular star compact with a k-in-countable base for some $k \in N$, then X is metrizable.*

Proof. Let $\mathcal{B}$ be a k-in-countable base of X for some $k \in N$.

We prove that $e(X) = \omega$. Suppose not and let $D = \{x_\alpha \colon \alpha < \omega_1\}$ be an uncountable closed discrete subset of X. Pick a $B_\alpha \in \mathcal{B}$ such that $B_\alpha \cap D = \{x_\alpha\}$ for each $\alpha < \omega_1$. Let $\mathcal{U}_0 = \{X \setminus D\} \cup \{B_\alpha \colon \alpha < \omega_1\}$. Since X is star compact, there is some compact subset K_0 of X such that $st(K_0, \mathcal{U}_0) = X$. It is not difficult to see that $B_\alpha \cap K_0 \neq \emptyset$ for each $\alpha < \omega_1$. The set K_0 is metrizable by Lemma 3.2, and hence separable. Let S_0 be a countable dense subset of K_0. There is some point $z_0 \in S_0$ such that z_0 belongs to uncountably many elements of $\{B_\alpha \colon \alpha < \omega_1\}$. Obviously, $z_0 \notin D$. Denote $\alpha_0 = \min\{\alpha < \omega_1 \colon z_0 \in B_\alpha\}$. Let $\mathcal{B}_0 = \{B_\alpha \colon z_0 \in B_\alpha,\ \alpha > \alpha_0\}$, $\mathcal{W}_0 = \{B_\alpha \setminus \{z_0\} \colon B_\alpha \in \mathcal{B}_0\}$, $\mathcal{U}_1 = \mathcal{W}_0 \cup (\mathcal{U}_0 \setminus \mathcal{B}_0) \cup \{B_{\alpha_0}\}$. For open cover $\mathcal{U}_1$ of X, there is some compact subspace K_1 of X such that $st(K_1, \mathcal{U}_1) = X$. Also by Lemma 3.2, we can pick a countable dense subset S_1 of K_1. Because elements of $\mathcal{W}_0$ are distinct, there is some point z_1 of S_1 such that z_1 belongs to uncountably many elements of $\mathcal{W}_0$. We know that $z_1 \neq z_0$ and $z_1 \notin D$ from the construction of $\mathcal{W}_0$. Denote $\alpha_1 = \min\{\alpha < \omega_1 \colon z_1 \in B_\alpha\}$. Let $\mathcal{B}_1 = \{B_\alpha \colon (B_\alpha \setminus \{z_0\}) \in \mathcal{W}_0, z_1 \in B_\alpha,\ \alpha > \alpha_1\}$, $\mathcal{W}_1 = \{B_\alpha \setminus \{z_0, z_1\} \colon B_\alpha \in \mathcal{B}_1\}$, $\mathcal{U}_2 = \mathcal{W}_1 \cup (\mathcal{U}_0 \setminus \mathcal{B}_1)) \cup \{B_{\alpha_j} \colon j = 0, 1\}$. Continue in this way, for each $i \leqslant k-1$ we can choose a compact subset K_i, a point z_i of K_i and a subset $\mathcal{B}_i$ of $\{B_\alpha \colon \alpha < \omega_1\}$ such that $|\{B_\alpha \in \mathcal{B}_i \colon \{z_0, \dots, z_i\} \subset B_\alpha\}| = \omega_1$. At step $k-1$, we can get a subset $\{z_0, \dots, z_{k-1}\}$ of X such that $\{z_0, \dots, z_{k-1}\}$ belongs to uncountably many elements of $\{B_\alpha \colon \alpha < \omega_1\}$, a contradiction. So $e(X) = \omega$.

By Lemma 3.3, we know that X is a regular Lindelöf space. Notice that any pseudocompact Lindelöf spaces is compact. By Lemma 3.2, the space X is metrizable. □

Corollary 3.5. *If X is a regular star compact with a 2-in-finite base, then X is metrizable.*

Remark 3.6. (1) It was asked in [11, Problem 4.8] that whether a star compact space is metrizable if it has a G_δ-diagonal. Since every space with a 2-in-finite base has a G_δ-diagonal, Corollary 3.5 can be regarded as a partial answer to this problem.

(2) It is known that countably compact spaces with point-countable bases are metrizable, while there exist pseudocompact spaces with point-countable bases which are not metrizable. Observe that star compactness is weaker than countable compactness but stronger than pseudocompactness so the following result gives new information about star compact spaces.

Corollary 3.7. *Any regular star compact space with a point-countable base is metrizable.*

Example 3.8. In Theorem 3.4 as well as in Corollaries 3.5 and 3.7, regularity cannot be weakened to Hausdorff property.

Proof. The space X was constructed in [10, Example 2.4.4] as follows:

Let $Y = \bigcup\{[0,1] \times \{n\} \colon n \in N\}$ and $X = Y \cup \{a\}$ where $a \notin Y$. Define a basis for a topology on X as follows. Basic open sets containing a take the form $\{a\} \cup \{[0,1) \times \{n\} \colon n \geqslant m\}$ where $m \in N$. Basic open sets about other points of X are the usual induced metric open sets.

It is easy to construct a 2-in-finite base in Y which, together with the local base at a forms a 2-in-finite base in X. However, it is proved in [10] and [9] that X is a second countable, star compact and non-metrizable Hausdorff space. □

Lemma 3.9. *If a regular countably compact space X has a k-in-countable weak base for some $k \in N$, then X is sequentially compact.*

Proof. Suppose that X is a regular countably compact space with a k-in-countable weak base $\mathcal{B}$ for some $k \in N$. If $\{x_n\colon n \in N\}$ is a countable infinite subset of X, then there is a subset of it which is not closed in X. Otherwise, $\{x_n\colon n \in N\}$ is a discrete closed subset of X, which contradicts the fact that X is countably compact. Without loss of generality, we may assume that $\{x_n\colon n \in N\}$ is not closed. Let $D = \overline{\{x_n\colon n \in N\}}$, then D is a countably compact closed subspace of X, and $\mathcal{B}|_D = \{B \cap D\colon B \in \mathcal{B}\}$ is a k-in-countable weak base of D. Since $\{x_n\colon n \in N\}$ is not closed in D, there is some point $y \in D \setminus \{x_n\colon n \in N\}$ such that $B'_y \cap \{x_n\colon n \in N\} \neq \emptyset$ for each $B'_y \in \mathcal{B}_y|_D$. It is easy to see that $|B'_y \cap \{x_n\colon n \in N\}| = \omega$ for each $B'_y \in \mathcal{B}_y|_D$. Therefore y has a countable local weak base since $\mathcal{B}|_D$ is a k-in-countable. This means that there is sequence $\{z_n\colon n \in N\}$ in $\{x_n\colon n \in N\}$ such that $z_n \to y$. $\square$

Lemma 3.10. *([5]) Every regular countably compact space with a point-countable weak base is metrizable.*

Recall that $\mathfrak{b} = \min\{|B|\colon B$ is an unbounded subset of ${}^{\omega}\omega\}$.

Lemma 3.11. *([12]) Compact Hausdorff weakly first countable spaces of size $< \mathfrak{b}$ are first countable.*

Theorem 3.12. *Suppose that X is a compact space with $|X| < \mathfrak{b}$. If X has a 2-in-finite weak base $\mathcal{B} = \bigcup\{\mathcal{B}_x\colon x \in X\}$ which consists of sequentially closed subsets of X, then X is metrizable.*

Proof. Let A be the set consisting of all the non-isolated points in X, and $A' = \{x \in X\colon$ there is a nontrivial sequence $\{x_n\colon n \in N\}$ such that $x_n \to x\}$. To prove that $A' = A$ assume that there is a point $z \in A \setminus A'$. Let $\mathcal{B}_z$ be a local weak base of z. Then $|\mathcal{B}_z| > \omega$ and $|B| \geqslant \omega$ for each $B \in \mathcal{B}_z$. Pick $B_1 \in \mathcal{B}_z$ and a nontrivial sequence $S_1 \subset B_1$. We can choose a point $x_1 \in A' \cap B_1$ since B_1 is sequentially closed and X is sequentially compact by Lemma 3.9. We have $x_1 \neq z$ since $z \in A \setminus A'$. There exists some $B_2 \in \mathcal{B}_z$ such that $B_2 \subset B_1$ and $x_1 \notin B_2$. Pick some sequence S_2 such that $S_2 \subset B_2$. We choose a point $x_2 \in A' \cap B_2$ and $x_2 \neq z$. Repeating this operation, we can get a sequence $\{x_n\colon n \in N\}$ and a family $\{B_n\colon n \in N\}$ of $\mathcal{B}_z$ such that $x_n \in B_n \cap A'$, $B_{n+1} \subset B_n$ for each $n \in N$ and $x_i \notin B_j$ if $j > i$. Then there is some point $x \in A'$ such that $x \in B_n$ for each $n \in N$, since X is sequentially compact and B_n is sequentially closed for each $n \in N$. This means that $|\{B \in \mathcal{B}\colon \{x, z\} \subset B\}| \geqslant \omega$, which is a contradiction. Therefore, $A' = A$. This fact means that A is weakly first countable.

In the proof Theorem 1.10 of [8], it is established that a weak neighborhood of a point must be a neighborhood of this point in a weakly first countable and Fréchet space. So $\mathcal{B}' = \{B^o \cap A\colon B \in \mathcal{B}\}$ is a 2-in-finite base of A by Lemma 3.11. Thus, A is metrizable by Lemma 3.2. Let S be a countable dense subset of A. We show that $|\bigcup\{\mathcal{B}_z\colon z \in A\}| = \omega$. It is obvious that $|\mathcal{B}_z| = \omega$ for each $z \in S$. There is some subsequence of S convergent to y for each $y \in A \setminus S$. $|\bigcup\{\mathcal{B}_z\colon z \in A \setminus S\}| = \omega$ since $\mathcal{B}$ is 2-in-finite. Let $\mathcal{W} = \bigcup\{\mathcal{B}_z\colon z \in A\} \cup \{\{x\}\colon x \in X \setminus A\}$. We can see that $\mathcal{W}$ is a point-countable weak base of X. Therefore, X is metrizable by Lemma 3.10. $\square$

Theorem 3.13. *Suppose that X is weakly countably tight (or a k-space) with a k-in-countable base $\mathcal{B}$ for some $k \in N$. Then X is strongly monotonically monolithic.*

Proof. If X is a k-space, then it is sequential by Lemma 3.2. The family $\mathcal{B}$ is point-countable at each non-isolated point by Lemma 3.1 if X is weakly countably tight or a k-space. Denote $I(X)$ the set of all isolated points in X. Let $\mathcal{B}_x = \{B \in \mathcal{B}\colon x \in B\}$ for each $x \in X \setminus I(X)$. For each $A \subset X$ we can assign an external base $\mathcal{O}(A)$ to the set $\overline{A}$ in such a way: $\mathcal{O}(A) = \{\{x\}\colon x \in I(X) \cap A\} \cup \{B \in \mathcal{B}_x\colon x \in (X \setminus I(X)) \cap A\} \cup \{B \in \mathcal{B}\colon H \subset B$ for some $H \subset X$ with $|H| = k\}$. It is not difficult to see that $\mathcal{O}(A)$ defined above satisfies the conditions (b) and (c) of the definition. We will prove next that $\mathcal{O}(A)$ also satisfies condition (a). We can see that $|\mathcal{O}(A)| \leqslant \max\{|A|, \omega\}$ from the construction of $\mathcal{O}(A)$ since $\mathcal{B}$ is k-in-countable and is point-countable at each non-isolated point of X. For each $x \in \overline{A}$ and each open set U of X with $x \in U$, if $x \in A$, we can easily choose some $H \in \mathcal{O}(A)$ such that $x \in H \subset U$; if $x \in \overline{A} \setminus A$, we can pick some $B \in \mathcal{B}_x$ such that $x \in B \subset U$. Finally, $B \in \mathcal{O}(A)$ since $|B \cap A| \geqslant \omega$. $\square$

Remark 3.14. (1) There is a space X with a 2-in-finite base, but it is not first countable [4, Example 1], hence not strongly monotonically monolithic. Thus, the condition "X is weakly countably tight (or a k-space)" in Theorem 3.11 cannot be omitted.

(2) Assuming Martin's Axiom and $\omega_2 < 2^{\omega_0}$, there is a normal Moore space with a 2-in-finite base that has no point-countable base [3, Theorem 4]. This space is strongly monotonically monolithic by Theorem 3.13, and hence we can consistently answer Question 1.3 negatively.

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